Quantum Macroeconomics Money Flow

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Introduction

Economics have begun only recently to translate its laws in the language of quantum statistics, approaching only specific areas of interests (like Baaquie, 2004; Orrell, 2018), leaving unaltered the groundings of mainstream economics. Schmitt (1972; 1984) suggested a complete reformulation of neoclassic macroeconomics, but he did it introducing the concepts of *wave distribution* and *transformation* only in a humanistic frame, leaving unaltered the mathematics underneath.

Here – on the basis of Rossi (2019-2020) – I suggest a basic reformulation of money theory, applying a slightly altered algebra of Dirac (1930), which I am giving account for (particularly in composition of matrices and vectors); and applying the basic principles of quantum mechanics coded by Feynman/Leighton/Sands (1963/2013), in order to explain how money flows through an economic system, meaning a circular model of money supply. That should explain why "natural" distributions of money (out of political control) tend to realize unfair or unsustainable solutions.

Keywords

Money, flow, supply, demand, vector, matrix, bra, ket, quantum, statistics, distributions, allocations, system, macroeconomics.

High powered money and liquidity states in quantum models

Quantum physics manage billions of particles in complex systems of matter, just like quantum economics manage billions of people and money in complex economic systems: a statistical approach describe the possibilities and the probabilities of an event to occur in a complex system, while it would be impossible to describe the specific behavior of a single – a specific – element of that system; that quantum being a particle or a person or money.

That way, I can think of a monetary system as a set of agents (quanta) who exchange different quantities of money (quanta), after a central agency put money into circulation.

[1] $$\langle L|H\rangle = M$$

I can think of a basic monetary system represented by equation #1 (called *inner product*): central agency (like a central bank) issues a certain quantity of high powered money (H) – or *base* – the "quantum state" vector $|H\rangle$, distributed to the agents in different ratios (listed as a column vector), and allocated in a final state of liquidity $\langle L|$ (a row vector listing other ratios); the two distributions resulting in the total amount of money supply (M) for the system. Sum of the ratios in each vector equals 1.

Dirac's algebra computes elements from left to right, and "it does not commute", meaning that $\langle L|H\rangle \neq |H\rangle\langle L|$ (more specifically: $\langle X|Y\rangle = A$, a number; $|X\rangle\langle Y| = \hat{A}$, a matrix). Feynman/Leighton/Sands (1963/2013) notice that I should "read" the "meaning" of formulas from right to left, $\langle L|H\rangle = M$ meaning that I "observe" M as the result of the initial state $|H\rangle$ "collapsing" in the final state $\langle L|$.

The initial state of money emission $|H\rangle$ represents a column vector (named *ket* in Dirac's algebra) listing the ratios of the total high powered money (or base money), H (the quanta of the system), allocated to every single agent (people being other quanta of the system). Thus, the column vector $|H\rangle$ lists N ratios or coefficients (N being the number of agents). E.g.: given $H = 18$, and $N = 3$, initial state $|H\rangle$ could list a triplet [0,3 0,6 0,1]. Total sum of N ratios must compute 1 in every vector.

On the other hand, the final state $\langle L|$ represents a row vector (named *bra* in Dirac's algebra) which represents the "collapse of the wave function": only one of all the possible allocations (discussed in the last chapter of this paper) makes all the other possibilities "collapse".

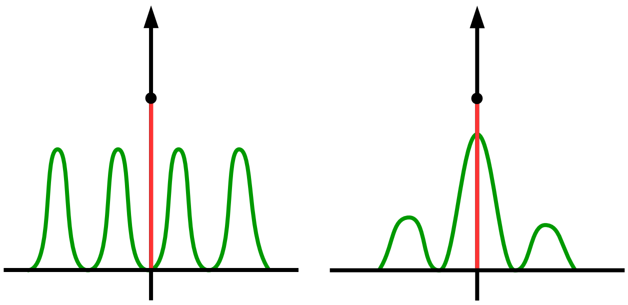

Fig. 1. *Two possible distributions of the same base.*

The two waves in figure #1 depict the value of two different distribution of money supply (M): rich agents get toward the crests of a wave, while poor agents get toward the troughs. The two vertical axes (y) of the diagrams in figure #1 measure the quantity of money, while the horizontal axes (x) list the different agents of the system: the red vertical lines (lying on both of the two vertical axes) depict the total amount of base money (H) *to be allocated* in the initial state $|H\rangle$, when all of the money lays still in the hands of the central agency; while every single wave depicts one possible evolution of distributions of money between the agents. There can be many kinds of waves: e.g., a horizontal line depicts an ideal scenario where everybody holds the same sum of money; a wave with all its troughs over the zero depicts a scenario where everybody has some sum of money; and so on. Central agency has no money in the wave depicted in the diagram on the left of figure #1, while it takes almost all of the money back in the wave depicted in diagram on the right.

The sum of the values recorded in a wave could be greater of the base money ($M > H$) if the transactions between the agents expand H, just like the mainstream theory of *money multiplier* states: $M = mH$ (its elementary formulation) meaning that money supply (M) depends on economic policies (denoted by multiplier m) acting on the high powered money (H). A higher quantity of crests in a wave implies a higher quantity of transactions (just like in the diagram on the left of figure #1), while a lower quantity of crests implies a lower quantity of transactions (just like in the diagram on the right of figure #1). Even the *multiplier theory* modeled by Keynes (1936) complies with that interpretation: $\Delta Y = k\Delta I$ (its elementary formulation) meaning that incremental investments (ΔI) expand the national production (ΔY) via the multiplier ($k = {}^{1}\!/_{1-c}$ its elementary formulation); so that an increase in c (the *marginal propensity to consume*, $c = {}^{\Delta C}\!/_{\Delta Y}$) expands k, which expands ΔY. That way, mainstream economics explains the reason why basic state $|H\rangle$ could allocate a liquidity state $\langle L|$, which determines a money supply M greater than the initial value H, via the transactions carried out on the basis of c.

Quantum theory of allocation – the wave distribution – summarizes the two mainstream multipliers in a complex theme, and it explains why centralization of assets and amount of transactions boost or break the processes that enhance the value of monetary base. Thus, knowing the structure of transactions $\langle L|H\rangle$, and the amount of population N, I could program an emission base H and an emission state $|H\rangle$ capable to determine the maximum value of M; or at least I can avoid the worst scenario in M.

Quantum transactions and allocations evolution

A final state $\langle L|$ can represent many number of allocations (not infinite number, as pointed out in the last chapter of this paper), depending on the behaviors taken by the agents between the initial allocation $|H\rangle$ and the final state $\langle L|$.

[2] $$\langle L|T|H\rangle = \langle L|T'\rangle\langle T|H\rangle$$

Equation #2 describes the evolution of the system as an initial allocation $|H\rangle$ collapsing in the final distribution $\langle L|$ via a series of transactions $T = |T'\rangle\langle T|$; T being a *matrix* resulting from the *outer product* $|T'\rangle\langle T|$, which represents the interactions between both public and private agents. More specifically, trades (consumption, savings and speculation) mediated by money allocate the initial state $|H\rangle$ to the final state $\langle L|$ via the allocation of intermediate states (T). Actions of the agents (government, families, and companies) in T "enhance" or "reduce" the initial base (H), defining money supply (M) of the economic system, as stated in equation #1.

Distribution of money to the agents (i.e., how much money gets every agent in a certain time) and frequency of the trades (i.e., how much money every agent reallocates to others) define the shape of a wave, thus the total value of money (M) in a system. Thus, I can visualize the circular flow of money like a series of matrices (T), following one another in time (or like a single matrix shifting its values in time): each row (and column) representing an agent in the system. Matrices record nominal money transfers, regardless of the reason why (transaction, saving or speculation, that flow via money): "money map" displays the money distribution flowing through the hands of agents.

		CREDITS		
	(**18**)	A	B	C
DEBTS	A	**6**	3	3
	B	2	**6**	2
	C	0	0	**6**

Tab. 1. *Example matrix.*

Instead of recording every intermediate vector, like $|T'\rangle$ and $\langle T|$ (and infinite others in the flow) in equation #2, I can keep account of transaction in T matrices like the one in example table #1. Matrix operators (T) describe how money flows through the agents, as a distribution of their interactions: that way quantum economics explains how economic activities create – or destroy – value. Matrix #1 visualizes a system with $H = 18$ fairly allocated to the three (only) agents (A, B, C) – the bold trace of the matrix matching H – so that each agent has $6L$ ($^{18}\!/_3 = {}^{H}\!/_{N}$) in a first stage $\langle T|$ of the matrix. Other cells record transactions in a second stage $|T'\rangle$: I subtract horizontally the debts from each initial allocation; while I add up the credits vertically to the initial allocations. In matrix #1, agent A pays $3L$ to agent B and pays $3L$ to agent C, while agent A receives $2L$ from agent B and receives $0L$ form agent C; in turn agent B pays $2L$ to agent C. Then I can record the final state of allocation ($2_A = 6_A–3_B–3_C+2_B$; $5_B = 6_B–2_A–2_C+3_A$; $11_C = 6_C+3_A+2_B$) in a new matrix (with trace always equal to H), or I can record the final state $\langle L|$ (listing final ratios allocated to agents). Precaution: matrices list sums of money and record always two stages of allocation; while vectors list ratios of initial allocation, $|L\rangle$, or final allocation, $\langle L|$.

Electronic money (like bank accounts, PayPal accounts, bitcoins and blockchains, etc.) could be used as infrastructure fitted for that system, as I will point out in the last chapter of this paper.

[3.1] $$\begin{bmatrix}0{,}33 & 0{,}34 & 0{,}33\end{bmatrix}\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}\begin{bmatrix}0{,}33\\ 0{,}34\\ 0{,}33\end{bmatrix}=2$$

[3.2] $$\begin{bmatrix}0{,}33 & 0{,}34 & 0{,}33\end{bmatrix}\begin{bmatrix}10 & 5 & 0\\ 0 & 4 & 1\\ 2 & 0 & 4\end{bmatrix}\begin{bmatrix}0{,}33\\ 0{,}34\\ 0{,}33\end{bmatrix}=3{,}10$$

[3.3] $$\begin{bmatrix}0{,}33 & 0{,}34 & 0{,}33\end{bmatrix}\begin{bmatrix}10 & 5 & 0\\ 0 & 4 & 1\\ 2 & 0 & 4\end{bmatrix}\begin{bmatrix}6 & 2 & 1\\ 3 & 10 & 0\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}0{,}33\\ 0{,}34\\ 0{,}33\end{bmatrix}=27{,}37$$

Examples above display differences in M – given the same initial fair allocation of $H = 18$ – resulting from stasis (none of the agents in equation #3.1 exchanges money, reducing $H = 18$ to $M = 2$); from four stages of transactions (equation #3.2 recording the transition from $|H\rangle$ to $[10_A \;\; 4_B \;\; 4_C]$, then to $[7_A \;\; 8_B \;\; 3_C]$ inside the matrix, then back again to $[6_A \;\; 6_B \;\; 6_C]$ in the bra); and from a six-stages flow (equation #3.3, computing $M > H$).

[4] $$\langle L|M\rangle = \langle M|L\rangle^*$$

The total value of money supply (M) can be (as indeed it is in reality) allocated to agents so that I can treat it like a state vector $|M\rangle$ or $\langle M|$. Here a general law of quantum mechanics states the circular idea expressed in equation #4: money supply (M) and money demand (L) are conjugated (the symbol * meaning *complex conjugate* of a state). Quantum economics shifts the mainstream idea of demand-supply equilibrium to the idea of a *circular relation*: money supply $|M\rangle$ transits toward a distribution coherent with the liquidity demand $\langle L|$ (on the left side of equation #4), just like the conjugate state of money demand $|L\rangle^*$ transits toward a conjugate state of money supply $\langle M|^*$ (on the right side of equation #4). Each state evolves to the other: an initial demand state transits toward a final supply state, that becoming an initial state for a final state in a new economic cycle or flow.

Quantum economics thinks of money as a *flow*, rather than a *stock*. Money created and destroyed (in cycles of transactions, savings and speculations) flows in a circular flux – its value *fluctuating* like a wave (see equations #2 and #4) via the mediation of banks – just like Schmitt (1972) and Cencini/Gnos/Rossi (2016) suggested: income and its purchasing power derive from the flux of payments mediated by money. Quantum economics observes a symmetry between high powered money (H) emitted by central banks and account money (M) emitted by commercial banks: any monetary emission state $|H\rangle$ (from whatever central agency) transits toward a distribution $\langle L|$ flowing through transactions, savings and speculation, that I can think of as matrices, as stated in equation #3.

Economic policies should define an emission state $|H\rangle$ and a public expenditure matrix (classical G) apt to allocate money in a "fair" way to the agents, maximizing the money's exchange value, which relies on exchange rates, on market access conditions, and on market efficiency conditions; thus relies on economic policies.

Quantum macroeconomics replicates quantum physics: both of them cannot state *who* will have *some* money or *where* a *certain particle* will be (given the size of one system), but they can describe the condition necessary for the search, and they can describe the probabilities of finding a particle, just like the conditions for money distribution. And different conditions of distribution – $|H\rangle$ and $\langle L|$ – compute different money values (M): they can crate wealth or poorness.

Descriptions supplied by previous equations replace the mainstream idea of money market *equilibrium* with the idea of *statistical distribution* of money because, *de facto*, classical (theoretical) equilibrium between demand quantity (L) and supply quantity (M) is impossible: the relation between demand and supply is always unstable in an economic system always flowing, where momentum (of transactions) continuously modifies the relativistic positions of (the states of) its components; who is demanding money on one hand is supplying that same money on the other hand. The idea of a *wave* depicts really good the idea of an ever-changing variable, just like (the value of) money is always passing through hands: as soon as I define the "demand", money behaves like "supply"; it is supplied to somebody who wants to spend it ("demanding" it), but therefore that somebody supplies that same money to somebody else, defining a new distribution state, acting another "demand" for that somebody else; and so on like stated in equation #3. The dual turnover cycle (demand-supply-demand) keeps going on continuously; series of money transitions – like $\langle L|T''|T'|T|H\rangle$ or $\langle L''|L'\rangle\langle L|H\rangle$ – change in fractions of seconds everyday; so that every distribution follows every emission in a *continuum* which restate the dual complementarity of states: $\langle L|M\rangle = \langle M|L\rangle^*$.

Static equilibrium is impossible because distribution and emission phases never stop: the wave changes continuously its frequency via T. Equation #5 computes those changes, or variation rates in allocations ($\dot{R}_i$) for each agent (i):

[5] $$\dot{R}=\frac{L_1-L_0}{L_0}$$

The system tends toward poorness, if transactions keep recording unfair allocation of money (although high powered money keeps constant), because agents excluded from distribution will not be spending money in future transactions; and the system tends toward poorness, if it records low rates of transactions.

[6] $$B = \sum_{a}^{n} \dot{R}_a$$

Equation #6 computes the sum of all the variation rates ($\dot{R}$): that way computing the trend (B) of the system, recording its impoverishment or its enrichment.

[7] $$M_{\mathrm{B}} = H + BH$$

Finally, equation #7 computes money supply (M) in a complete cycle of the account matrices flow: expansion or contraction induced by the variation rate ($\pm B$) operating on the initial money base (H).

Quantum distributions and fairness

Macroeconomics usually measure competitiveness relying on *per capita* income ($^{Y}/_{N}$), but that measurement could be biased because it results in an *average value*: stating that every agent owns the fraction $^{Y}/_{N}$ of national income is obviously erroneous because it misrepresent reality of facts, knowing that there are poor agents and rich agents – there are different degrees of allocation – in one system.

A distribution vector $\langle L|$ gathers clearer information about the economic potential of a system because it defines the economic possibilities of the agents in terms of their potential transactions, just like a matrix represents their interactions: two different distributions of the same base define different conditions for the growth of a system. E.g., distribution $\langle L|H\rangle = 0_{\mathrm{A}}+1_{\mathrm{B}}+17_{\mathrm{C}}$ ($H = 18$) has a lower growth potential compared to distribution $\langle L|H\rangle = 6_{\mathrm{A}}+7_{\mathrm{B}}+5_{\mathrm{C}}$ ($H = 18$), while both of them measure the same *per capita* income ($^{18}/_{3} = 6$). Theoretically the system in the first example tends to stop – because the two “poor” agents (0_{A} and 1_{B}) easily tend to stop their trading possibilities – while the other system allows agents to employ their incomes in a greater amount of transactions and allocations. In the former example, only the “rich” agent (17_{C}) has the power to allocate wealth through the system, while in the latter example every agent can do it.

Here I note a paradox, which I should explain (in another paper) in terms of economic policy: extreme unfair distributions, like (for instance) $\langle L|H\rangle = 0_{\mathrm{A}}+18_{\mathrm{B}}+0_{\mathrm{C}}$ ($H = 18$), show high values in M (in terms of equation #1), though those extreme unfair distributions compute low variation rates (in terms of equation #5); as I can see in the following static example equation #3.4 ($M = H$), compared to fair and static equation #3.1 ($M < H$), and compared to fair and dynamic equation #3.2 ($M < H$):

[3.4] $$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 18$$

Nevertheless I note an assumption: quanta of the system – people and money – define possibilities and trend of the system.

Distribution possibilities are finite (discrete) because money quantizes wealth (money is the minimum unit of wealth) and people quantize population (a person is the minimum unit of population): thus the two (discrete) quanta of macroeconomics account for discrete distributions, i.e. discrete combinations of people and money. For instance, I cannot *observe* 3,2 units of money in the hands of 6,7 agents, but I observe 3 units of money in the hands of 7 agents. Thus I can state that one same monetary base (e.g., $H = 18$) can be allocated between a certain number of agents (e.g., $N = 3$) in the limits of the formula derived from Bernoulli (1713) as the number (D) of *combinations with repetition*:

[8] $$D=\frac{(H+N-1)!}{H!(N-1)!}$$

Equation #8 computes ${}^{20!}/_{18!2!} = 190$ possible distributions (given $H = 18$, and $N = 3$), where ideal distribution $\langle L|H\rangle = 6_A+6_B+6_C$ – or *per capita* income (${}^{18}/_{3} = 6$), the most fair distribution – is also the less likely distribution to occur (there is only 1 possibility out of 190 that H would be allocated in a perfect fair state); and even the most unfair distribution (all the money in the hands of only one agent) has a few chances to occur (there are just 3 possibilities out of 190; everything in the hands of agent A or in the hands of agent B or in the hands of agent C). But there is another distribution, just a bit more fair than the latter, which is more likely to occur: for instance, there are 15 different combinations out of 190 for the *80-20 distribution* to occur (*Pareto principle*, 1897; restated by Juran, 1951), meaning that 80% of money is allocated to 20% of agents – Piketty (2013) noticed a 90-10 distribution in the late Nineties of 20th century. Other unfair distributions (e.g., 70% in the hands of 1 single agent, and the rest to others) occur with commonly high probabilities. That meaning: cluttered (disorganized) distributions, allocating the average quantity (${}^{H}/_{N}$) in the hands of a single agent (and the rest to others), are as likely to occur as the organized distributions, allocating everything in the hands of one agent.

That trend – pushing toward a wealth gap between agents – seems to reject the "law of entropy" (or second law of thermodynamics), which states that particles (quanta) tends to organize disordered states of matter (like $\langle L|H\rangle = 6_A+7_B+5_C$ in money terms), instead of ordered states (like $\langle L|H\rangle = 0_A+0_B+18_C$ or the Pareto distribution $\langle L|H\rangle = 2_A+2_B+14_C$). Actually, perfect economic fairness (or disordered states) and perfect economic unfairness (or ordered states) tend to coincide in money quantum theory (e.g., ${}^{1}/_{190} \approx {}^{3}/_{190}$ in previous examples).

Economics habitually think of composition of income (Y) as a deterministic system: linear equations, defining macroeconomic variables, are intrinsically deterministic (e.g., equation $Y = C+I+G–T+X–M$ states that variations on the right side of equation vary Y on the left side). But money distribution relies even on chances, just like Chesney/Scott (1989), Biondo/Pluchino/Rapisarda (2014) and other financial markets analysts stated. E.g., inception of new agents in a system enhances unfairness, expanding the number of possibilities computed by equation #8 (those 190 combinations in the previous example inflate to 1330, if I compute 4 agents instead of 3), consequently expanding the number of unfair probabilities.

Modern information technologies can take account of trade matrices for quantum analysis of economic systems: banks and other intermediaries (like PayPal, bitcoin chains, etc.) can do it, just like Internet players are recording and analyzing big data.

Actually Congress of USA worried about Facebook's proposition to develop its own e-money (*Libra*): the idea that a central private agency could control and distribute money worries public institutions, insisting on claiming specific policies and regulations, by which they can constraint free chance, on one hand, and free choice, on the other hand.

Equation #8 states that some relations between quantity of money and number of agents foster fairness, while other numbers foster unfairness. E.g., given $H = 6$ and $N = 3$, there are 28 total possible combinations of allocations, out of which only 10 combinations avoid at least one pauper agent (getting zero): $[2_A+2_B+2_C]$, $[2_A+1_B+3_C]$, $[1_A+2_B+3_C]$, $[2_A+3_B+1_C]$, $[1_A+3_B+2_C]$, $[3_A+2_B+1_C]$, $[3_A+1_B+2_C]$, $[4_A+1_B+1_C]$, $[1_A+4_B+1_C]$, $[1_A+1_B+4_C]$; while other 18 combinations imply paupers, constraining trades possibilities of the system.

[9] $$E=\frac{F}{D}$$

Equation #9 computes the *potential trades index* (E) as the relation between number of distributions avoiding pauper agents (F, 10 combinations in previous example) and number of total distributions (D, 28 combinations in the example): $^{10}/_{28} \approx 0{,}36$. But E drops to $0{,}25 = {}^{7}/_{28}$, if I think of excluding three unfair distributions: $[4_A+1_B+1_C]$, $[1_A+4_B+1_C]$, $[1_A+1_B+4_C]$. And E drops to $0{,}18 \approx {}^{10}/_{55}$, if I expand $H = 9$ (instead of $H = 6$), with 55 total possible combinations, out of which I can select (via a specific policy) only 10 fair states: $[3_A+3_B+3_C]$, $[3_A+2_B+4_C]$, $[2_A+3_B+4_C]$, $[2_A+4_B+3_C]$, $[3_A+4_B+2_C]$, $[4_A+2_B+3_C]$, $[4_A+3_B+2_C]$, $[5_A+2_B+2_C]$, $[2_A+5_B+2_C]$, $[2_A+2_B+5_C]$. And index E drops to $0{,}14 \approx {}^{9}/_{66}$, if I expand $H = 10$; given that I can compute that same index ($0{,}14 \approx {}^{3}/_{21}$) on the basis of $H = 5$, with $D = 21$ total combinations, out of which 3 combinations exclude pauper agents, $[1_A+1_B+3_C]$, $[1_A+3_B+1_C]$, $[3_A+1_B+1_C]$.

[10] $$P=1+\sum_{a}^{m}\frac{N!}{\prod_{j}^{n} j_a!}$$

Equation #10 computes the number of *possible fair distributions* (P) – a partition of set D – like the sum of *permutations with repetition*: given n agents holding one same distribution, which I select as a fair distribution – thus I need an economic policy in order to evaluate fairness –, adding the only (1) possibility that represents *per capita* distribution ($^{H}/_{N}$). That way, in previous example ($H = 10$, $N = 3$) distribution $[5_A+3_B+2_C]_a$ computes $^{3!}/_{1!\times1!\times1!} = 6$ permutations: only agent A (1!) holds $5L$, only agent B holds $3L$, only agent C holds $2L$. While distribution $[4_A+3_B+3_C]_b$ computes $^{3!}/_{1!\times2!} = 3$ permutations: both agents B and C (2!) hold $3L$. That way, summation of permutations (computed via equation #10) – excluding unfair distributions like $[9_A+0_B+1_C]$ – computes $P = 10$ (i.e., 1+6+3).

Dividing P by D, I can compute the global *fairness index* ($0{,}14 \approx {}^{9}/_{66}$) $Z = {}^{P}/_{D}$ via equation #11:

[11]
$$Z=\frac{1+\sum_{a}^{m}\frac{N!}{\prod_{j}^{n} j_a!}}{\frac{(H+N-1)!}{H!(N-1)!}}$$

That meaning I could program an efficient money supply (*M*), programming money emission (*H*) on the basis of index *Z*, which in turn is based on the two “natural” quanta of the system: money (*H*) and people (*N*).

References

Baaquie B.E. (2004), *Quantum Finance: Path Integrals and Hamiltonians for Options and Interest Rates*, Cambridge, University Press.

Bernoulli J. (1713), *Ars conjectandi. Opus posthumum*, Basel, impensis Thurnisiorum Fratrum.

Biondo A.E., Pluchino A., Rapisarda A. (2014), *Micro and macro benefits of random investments in financial markets*, “Contemporary Physics”, v. 55, n. 4, pp. 318-334.

Cencini A., Gnos C., Rossi S. (2016), *Quantum macroeconomics: A tribute to Bernard Schmitt*, “Cuadernos de Economía”, n. 39, pp. 67-75.

Chesney M., Scott L. (1989), *Pricing European Currency Options: A Comparison of the Modified Black-Scholes Model and a Random Variance Model*, “Journal of Financial and Quantitative Analysis”, v. 24, n. 3, pp. 267-284.

Dirac P.A.M. (1930), *The principles of quantum mechanics. Third edition*, Oxford, University Press.

Feynman R., Leighton R., Sands M. (1963), *The Feynman Lectures on Physics. 3 vol.*, Boston, Addison-Wesley; (2011) *New Millennium Edition*, New York, Hachette; (2013) *The Feynman Lectures Website*, California Institute of Technology, www.feynmanlectures.caltech.edu/index.html.

Juran J.M. (1951), *Quality Control Handbook*, New York, McGraw-Hill.

Keynes J.M. (1936), *The General Theory of Employment, Interest and Money*, New York, Macmillan.

Orrell D. (2018), *Quantum Economics: The New Science of Money*, London, Icon.

Pareto V. (1897), *Course d’économie politique*, Lausanne, Rouge.

Piketty T. (2013), *Le capital au XXIe siècle*, Paris, du Seuil.

Rossi O.D. (2019-2020), *Pensiero magico e scientifico: un'epistemologia dei saperi occidentali tra fisica, economia e antropologia*, Roma, IFOR, www.iformediate.-com/articoli-liberi/pensiero-magico-scientifico.pdf.

Schmitt B. (1972), *Macroeconomic Theory: A Fundamental Revision*, Albeuve, Castella.

Schmitt B. (1984), *Inflation, chômage et malformations du capital: macroéconomie quantique*, Paris-Albeuve, Economica-Castella.